Exotic Heavy Hadrons from QCD with Static Quarks

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ExoHad
Exotic Hadrons Topical Collaboration

QCD Degrees of Freedom and Hadrons

The QCD Lagrangian

$$\mathcal{L}_{\text{QCD}} = -\frac{1}{4}F^{A}_{\mu\nu}F^{\mu\nu}_{A} + \sum_{\text{flavors}} \bar{q}_a(i\gamma^\mu\partial_\mu - m)_{ab}q_b$$

Isolated states of QCD must be color singlets!

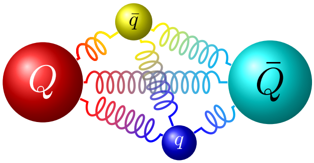
- $q\bar{q}$: quark-model mesons
- qqq: quark-model baryons
- other: exotic hadrons

Talk focused on isospin-0 double-heavy mesons

- $Q\bar{Q}$: quarkonia
- $Qg\bar{Q}$: hybrids
- $Q\bar{q}q\bar{Q}$: tetraquarks

Born-Oppenheimer Approximation for QCD

K.J. Juge, J. Kuti and C.J. Morningstar 1999

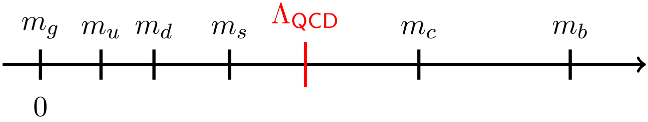


Heavy degrees of freedom

- heavy quarks

Light-QCD fields

- gluons
- light quarks



The Born-Oppenheimer Hamiltonian

Expansion in powers of $1/m$

$$H_{\mathsf{BO}}(\vec{r},\vec{p}) = H_{\mathsf{static}}(\vec{r}) + \frac{p^2}{m} + \dots$$

Leading order ($m \to \infty$): the static limit

$$H_{\mathsf{static}}(\vec{r}) = \sum_n |\zeta_n(\vec{r})\rangle \, V_n(r) \, \langle\zeta_n(\vec{r})|$$

- n Born-Oppenheimer quantum numbers
- $V_n(r)$ energy levels of light QCD with static $Q, \bar{Q}$ at distance r
- $|\zeta_n(\vec{r})\rangle$ eigenstates of light QCD with static $Q, \bar{Q}$ at $+\vec{r}/2, -\vec{r}/2$

Matching with Lattice QCD

Correlation matrix in light QCD with static $Q, \bar{Q}$ at $+\vec{r}/2, -\vec{r}/2$

$$C_{ij}(r,\tau,\tau_0) = \langle 0|\mathcal{O}_i(\vec{r},\tau)U(\tau,\tau_0)\mathcal{O}_j^\dagger(\vec{r},\tau_0)|0\rangle$$

The correlation matrix $\mathbf{C}$ can be calculated using lattice QCD.

QCD	quantity that is determined	B-O
$\mathbf{C}$ eigenvalues at large τ	static energy levels	$V_n(r)$
$\mathbf{C}$ eigenvectors at large τ	transition rates between levels	$\lvert\zeta_n(\vec{r})\rangle$

Truncation to N channels

N eigenvalues and eigenvectors $\rightarrow$ truncated B-O approximation

The Born-Oppenheimer Symmetries

The static $Q\bar{Q}$ break

- rotations,
- parity,
- charge-conjugation,

down to

- cylindrical symmetries,
- combined CP symmetry.

The quantum numbers are not

J angular momentum,

P parity,

C charge-conjugation,

but rather

λ angular momentum projection on the $Q\bar{Q}$ axis,

η (g or u) $CP = +$ or $-$.

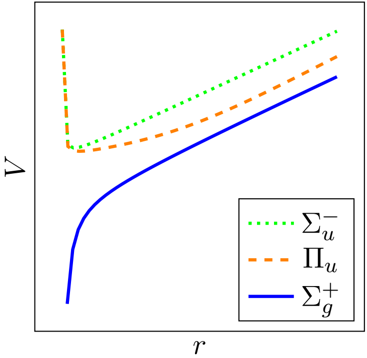
Heavy-quark spin symmetry

Static energy levels are independent of the heavy-quark spins.

Static Energy Levels of Pure $SU(3)$ Gauge Theory

K.J. Juge, J. Kuti and C.J. Morningstar 1999

S. Capitani, O. Philipsen, C. Reisinger, C. Riehl and M. Wagner 2019



Π_u, Σ_g^+: hybrid potentials

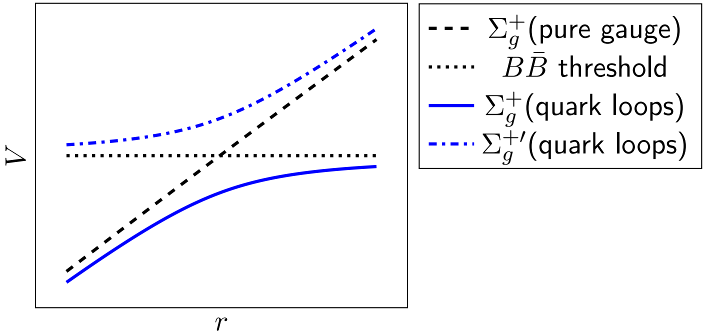
- $r \to 0$: 1^{+-} gluelump
- $r \to \infty$: $N = 1, 3$ string

Σ_g^+: quarkonium potential

- $r \to 0$: 0^{++} vacuum
- $r \to \infty$: $N = 0$ string

Σ_g^+ Potentials with String Breaking

G.S. Bali, H. Neff, T. Düssel, T. Lippert and K. Schilling 2005
J. Bulava, B. Hörz, F. Knechtli, V. Koch, G. Moir, C. Morningstar and M. Peardon 2019



String breaking couples different Born-Oppenheimer potentials!

Diabatic Born-Oppenheimer Approximation

W. Lichten 1963; F.T. Smith 1969

Adiabatic Schrödinger equation

$$-\frac{1}{m}\left(\vec{\nabla} + \vec{\mathbf{\Pi}}(\vec{r})\right)^2 \mathbf{\Psi}(\vec{r}) + \mathbf{V}_{\text{diag}}(r)\mathbf{\Psi}(\vec{r}) = E\mathbf{\Psi}(\vec{r})$$

transitions proceed through nonadiabatic coupling matrix $\vec{\mathbf{\Pi}}(\vec{r})$

unitary transformation

Diabatic Schrödinger equation

$$-\frac{\nabla^2}{m}\mathbf{\Psi}(\vec{r}) + \mathbf{V}(\vec{r})\mathbf{\Psi}(\vec{r}) = E\mathbf{\Psi}(\vec{r})$$

transitions proceed through diabatic potential matrix $\mathbf{V}(\vec{r})$

From Static Quarks to Duoble-Heavy Hadrons

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- The static energy levels with Born-Oppenheimer quantum numbers η, λ are the eigenvalues of a matrix $\mathbf{G}^{\eta,\lambda}(r)$ that solely depends on the distance r between Q and $\bar{Q}$.

- The diabatic potential matrix that depends on the relative position $\vec{r}$ of Q and $\bar{Q}$ is a linear combination of the matrices $\mathbf{G}^{\eta,\lambda}(r)$ for different values of λ,

$$V^{\eta}_{i,\sigma;i',\sigma'}(\vec{r}) = \sum_{\lambda} D^{j_i}_{\sigma,\lambda}(\varphi,\theta,\psi) D^{j_{i'}}_{\sigma',\lambda}(\varphi,\theta,\psi)^* G^{\eta,\lambda}_{i,i'}(r),$$

where the angular dependence is governed by Wigner D-matrix elements.

Diabatic Schrödinger Equation

- The introduction of the motion of the heavy quarks promotes the Born-Oppenheimer symmetries to the symmetries of QCD.
- The potential matrix for each J^{PC} is determined by:
  - angular-momentum coefficients;
  - functions of r calculable using lattice QCD;
  - threshold and reduced-mass corrections.

Radial potential for $Q\bar{Q}$ and S-wave-meson pairs with $J^{PC} = 1^{++}$

$$\begin{pmatrix} V_{Q\bar{Q}}(r) & \frac{1}{\sqrt{3}}g(r) & \frac{1}{\sqrt{6}}g(r) & \frac{1}{\sqrt{2}}g(r) \\ \frac{1}{\sqrt{3}}g(r) & -\frac{\Delta}{2} & 0 & 0 \\ \frac{1}{\sqrt{6}}g(r) & 0 & -\frac{\Delta}{2} & 0 \\ \frac{1}{\sqrt{2}}g(r) & 0 & 0 & \frac{\Delta}{2} \end{pmatrix}$$

Spectrum and Decays

Double-heavy hadrons

Are associated with poles of the S-matrix for the scattering of heavy-hadron pairs.

S-matrix

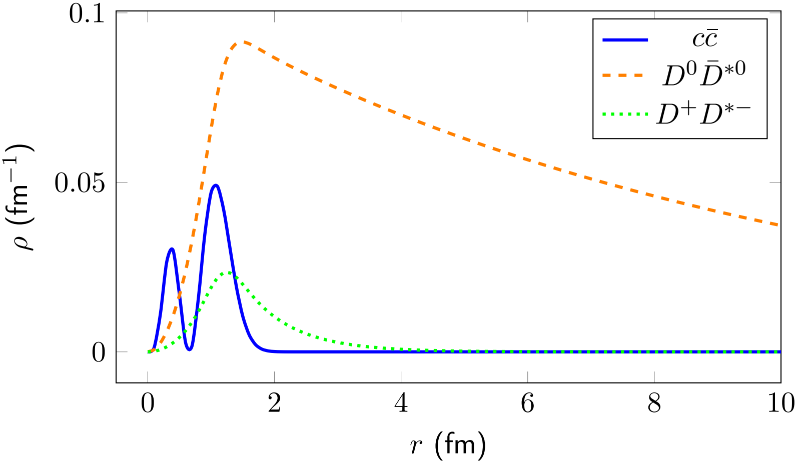
Can be calculated nonperturbatively by solving the Schrödinger equation for coupled $Q\bar{Q}$ and heavy-hadron-pair channels.

Decay selection rules

Can be determined without any input from lattice QCD using the Born-Oppenheimer symmetries.

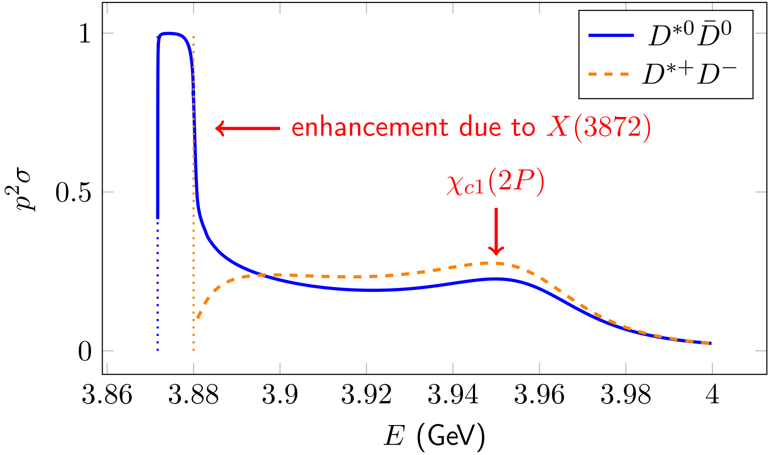
Mixing of Compact and Molecular States

RB and P. González 2022



Missing Conventional States

RB and P. González 2023



Decays of Lowest Hybrids into Pairs of S-Wave Mesons

RB 2024

Multiplet	J^{PC}		Potential
H_1	1^{--}	$(0,1,2)^{-+}$	Π_u/Σ_u^-
H_2	1^{++}	$(0,1,2)^{+-}$	Π_u
H_3	0^{++}	1^{+-}	Σ_u^-
H_4	2^{++}	$(1,2,3)^{+-}$	Π_u/Σ_u^-
H_5	2^{--}	$(1,2,3)^{-+}$	Π_u

Decays of Lowest Hybrids into Pairs of S-Wave Mesons

RB 2024

	Multiplet	J^{PC}		Potential
	H_1	1^{--}	$(0,1,2)^{-+}$	Π_u/Σ_u^-
forbidden	H_2	1^{++}	$(0,1,2)^{+-}$	Π_u
	H_3	0^{++}	1^{+-}	Σ_u^-
	H_4	2^{++}	$(1,2,3)^{+-}$	Π_u/Σ_u^-
forbidden	H_5	2^{--}	$(1,2,3)^{-+}$	Π_u

Decays of Lowest Hybrids into Pairs of S-Wave Mesons

RB 2024

	Multiplet	J^{PC}		Potential
allowed	H_1	1^{--}	$(0,1,2)^{-+}$	Π_u/Σ_u^-
forbidden	H_2	1^{++}	$(0,1,2)^{+-}$	Π_u
allowed	H_3	0^{++}	1^{+-}	Σ_u^-
	H_4	2^{++}	$(1,2,3)^{+-}$	Π_u/Σ_u^-
forbidden	H_5	2^{--}	$(1,2,3)^{-+}$	Π_u

Allowed decays of H_1, H_2, H_4 contradict the conventional wisdom that hybrids do not decay into pairs of S-wave mesons.

- The spectrum and decays of double-heavy hadrons can be studied *ab initio* using the diabatic Born-Oppenheimer approximation for QCD.

- Many potentials and mixing effects left to calculate before detailed predictions of the spectrum are available.

- Some general features are already quite clear:
  - mixing of different constituents is a crucial part of the problem;
  - coupling with hadron-pair thresholds is particularly relevant;
  - many exotic states should be expected to be quite broad.